

# Trigonometric Equivalences

## Angles vs. Signs

$$\begin{aligned}\cos -\theta &= \cos \theta \\ \sin -\theta &= -\sin \theta\end{aligned}$$

## Periods

$$\begin{aligned}\cos \theta &= \sin\left(\theta + \frac{\pi}{2}\right) \\ \cos n\pi &= \sin\left(n\pi + \frac{\pi}{2}\right)\end{aligned}$$

## Common Values

$$\begin{aligned}\cos \frac{\pi}{6} &= \frac{\sqrt{3}}{2} \\ \sin \frac{\pi}{6} &= \frac{1}{2} \\ \tan \frac{\pi}{6} &= \frac{1}{\sqrt{3}}\end{aligned}$$

## Reciprocal Relationships

$$\begin{aligned}\csc \theta &= \frac{1}{\sin \theta} \\ \sec \theta &= \frac{1}{\cos \theta} \\ \cot \theta &= \frac{1}{\tan \theta}\end{aligned}$$

## Quotient Relationships

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \cot \theta &= \frac{\cos \theta}{\sin \theta}\end{aligned}$$

## Pythagorean Relationships

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ 1 + \tan^2 \theta &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$$

## Addition Formulas

$$\begin{aligned}\sin(\alpha + \beta) &= \sin \alpha \cos \beta + \cos \alpha \sin \beta \\ \cos(\alpha + \beta) &= \cos \alpha \cos \beta - \sin \alpha \sin \beta \\ \tan(\alpha + \beta) &= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}\end{aligned}$$

## Subtraction Formulas

$$\begin{aligned}\sin(\alpha - \beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ \cos(\alpha - \beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ \tan(\alpha - \beta) &= \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}\end{aligned}$$

## Double-Angle Formulas

$$\begin{aligned}\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1 \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha}\end{aligned}$$

## Half-Angle Formulas

$$\begin{aligned}\sin \frac{1}{2}\theta &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ \cos \frac{1}{2}\theta &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{1}{2}\theta &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}\end{aligned}$$

## Products of Sines and Cosines

$$\begin{aligned}\sin \alpha \cos \beta &= \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)] \\ \cos \alpha \sin \beta &= \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)] \\ \cos \alpha \cos \beta &= \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)] \\ \sin \alpha \sin \beta &= \frac{1}{2}[\cos(\alpha + \beta) - \cos(\alpha - \beta)]\end{aligned}$$

## Sum and Difference of Sines and Cosines

$$\begin{aligned}\sin \alpha + \sin \beta &= 2 \sin \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta) \\ \sin \alpha - \sin \beta &= 2 \cos \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta) \\ \cos \alpha + \cos \beta &= 2 \cos \frac{1}{2}(\alpha + \beta) \cos \frac{1}{2}(\alpha - \beta) \\ \sin \alpha + \sin \beta &= 2 \sin \frac{1}{2}(\alpha + \beta) \sin \frac{1}{2}(\alpha - \beta)\end{aligned}$$

## Law of Sines

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ \frac{\sin A}{a} &= \frac{\sin B}{b} = \frac{\sin C}{c}\end{aligned}$$

## Law of Cosines

$$\begin{aligned}a^2 &= b^2 + c^2 - 2bc \cos A \\ b^2 &= a^2 + c^2 - 2ac \cos B \\ c^2 &= a^2 + b^2 - 2ab \cos C\end{aligned}$$