Complex Variables

De Moivre

$$
\begin{aligned}
z &= x + iy = e^{i\theta} = \cos\theta + i\sin\theta \\
\bar{z} = \frac{1}{z} &= x - iy = e^{-i\theta} = \cos\theta - i\sin\theta \\
e^{i\frac{\theta}{n}} &= \cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}
\end{aligned}
$$

Powers

$$
\begin{aligned}
(\cos\theta + i\sin\theta)^n &= \cos n\theta + i\sin n\theta \\
(\cos\theta + i\sin\theta)^{-n} &= \cos n\theta - i\sin\theta \\
2\cos n\theta &= z^n + z^{-n} \\
2i\sin n\theta &= z^n - z^{-n}
\end{aligned}
$$

Trigonometric

$$
\begin{aligned}
\Im z = \sin\theta &= \frac{e^{i\theta} - e^{-i\theta}}{2i} = \frac{1}{2i}\left(z - \frac{1}{z}\right) \\
\Re z = \cos\theta &= \frac{e^{i\theta} + e^{-i\theta}}{2} = \frac{1}{2}\left(z + \frac{1}{z}\right) \\
\tan\theta &= \frac{\sin\theta}{\cos\theta}
\end{aligned}
$$

Hyperbolic

$$
\begin{aligned}
\sinh\theta &= \frac{e^{\theta} - e^{-\theta}}{2} \\
\cosh\theta &= \frac{e^{\theta} + e^{-\theta}}{2} \\
\tanh\theta &= \frac{\sinh\theta}{\cosh\theta}
\end{aligned}
$$

Complex Equivalences

$$
\begin{aligned}
\sin iz &= i\sinh z \\
\sinh iz &= i\sin z
\end{aligned}
$$

$$
\begin{aligned}
\cos iz &= \cosh z \\
\cosh iz &= \cos z \\
\cosh x = k \quad &\rightarrow \quad x = \ln(k \pm \sqrt{k^2 - 1}), \text{where}(k > 1)
\end{aligned}
$$

Hyperbolic Identities

$$
\begin{aligned}
\cosh(a + b) &= \cosh a \cosh b + \sinh a \sinh b \\
\cosh z &= \cosh x \cos y + i\sinh x \sin y
\end{aligned}
$$

Complex Roots

$$
\begin{aligned}
\omega &= z^{\frac{1}{n}} = r^{\frac{1}{n}}(\cos\theta + i\sin\theta)^{\frac{1}{n}} = r^{\frac{1}{n}} e^{i\frac{\theta}{n}} \\
&= r^{\frac{1}{n}}\left(\cos\frac{\theta + 2\pi k}{n} + i\sin\frac{\theta + 2\pi k}{n}\right)
\end{aligned}
$$

Roots of Unity

$$1 = \cos 2\pi k + i\sin 2\pi k = e^{i2\pi k}$$

$$1, \alpha, \alpha^2, \ldots, \alpha^{n-1} \quad \text{with } \alpha = \exp\left(\frac{2\pi i}{n}\right)$$